Mass and Magnetic Dipole Shielding against Electrons of the Artificial Radiation Belt

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Computations of magnetic shielding system masses required for protection against electrons of various energies have been performed. We have also obtained a comparison of the required shielding masses for a space vehicle traversing circular orbits through the anomaly in the artificial radiation belt (ARB) at altitudes of 200, 400, 600, and 800 km and inclined at 30° to the earth's equatorial plane: first with a magnetic superconducting solenoidal shielding system approximated by a point dipole and then with a two-layer composite material shielding system consisting of an outer layer of aluminum and an inner layer of lead. The aluminum thickness equaled the practical range of 10-Mev electrons, and the lead was used to attenuate the bremsstrahlung to specified skin dose rates. The averaged electron flux in ARB, used in estimating dose, was computed to be sufficiently accurate for missions not shorter than 100 hr and more accurate for longer missions. The material shielding mass, primarily designed for protection when appreciable bremsstrahlung occur, was at best (no lead) an order of magnitude larger than that of the magnetic shielding system. The mass of the cryogenic system was neglected, being a fraction of the combined mass of the solenoid and support structure.

Nomenclature

l = solenoid length
r = solenoid radius
m_w = solenoid mass
N = number of turns of the superconducting wire
ρ_w = density of the superconducting wire
d_w = diameter of the superconducting wire
I = current in the wire
H = magnetic field at the center of the solenoid
M = magnetic dipole moment
V = inner forbidden volume
R_p = practical range of electrons of 10 Mev
m_{st}' = mass of the containment shell
ρ_{st}/S_y = density-to-yield strength ratio of the containment shell
M_t = material shielding mass
R = dimensionless parameter
L = dimensionless parameter
M_{st}' = dimensionless parameter

Introduction

A COMPARISON is made of the relative masses required for an aluminum-lead shield vs a superconducting solenoidal shield for specific cases where the manned space vehicle is in any one of four practical circular orbits passing through the high electron flux region of the artificial radiation belt. The toroidally shaped totally forbidden volume of the dipole is compared with one of equal magnitude enclosed by a spherical shell of material. For the purposes of the calculations that follow, we assume that the electrons of the artificial radiation belt can be characterized by an omnidirectional flux of electrons with an energy distribution given by the fission-energy spectrum of Carter et al.[1]

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Magnetic Shielding Mass

Here we will obtain the masses of superconducting magnetic solenoid systems required for protection against electrons of energies up to 10 Mev for a range of shielded volumes utilizing the point-dipole analysis for the inner forbidden volume. For this analysis to be valid, the shielded volume should engulf the solenoid and its supporting masses. Otherwise, one would have to account for the x rays produced by the electrons within the system. We shall utilize an optimizing procedure to calculate the minimum mass for the containment shell. It will be shown that with proper choice of units the relationship between the mass for the containment shell and the dimensions of the superconducting solenoid can be made independent of the magnetic field parameters. Furthermore, this relationship defines a minimum value for the mass. We start by adopting the following model for the superconducting system.[2]

1) The solenoid is a right circular cylinder of radius r, length l, and mass m_w, comprising N turns of superconducting niobium-tin alloy wire of density $\rho_w = 8$ g/cm^3, and diameter $d_w = 0.015$ cm.

2) A cylindrical containment shell of mass m_{st}' made of a titanium alloy (Ti–6Al–4V ELI) fits closely about the windings and is utilized to prevent explosive expansion of the energized solenoid. The shell is assumed to have the same radius and length as the solenoid and has a high density-to-yield-strength ratio at 4°K of $\rho_{st}/S_y = 2.46 \times 10^{-10}$ g/erg.

3) Kash and Tooper[2, 3] show that, for larger shielded volumes, the mass of the thermal insulation and cryogenic systems will be small as compared to the combined mass of the windings and containment structure. Hence, we feel it reasonable to neglect the mass of thermal insulation and also to make no allowance for changes in structural configuration that would be required for coolant ducts in the solenoid. We further neglect the mass of the cryogenic equipment necessary to produce the low temperatures and compensate for heat leaks into the system. Finally, we neglect the possibility of reducing the magnitude of the structural mass by integrating it with the walls of the space vehicle.

Where compressive stresses are taken into account, the mass of the structural containment shell given by Refs. 3 and 4 is

$$m_{st}' = \frac{4M^2\rho_{st}(1+\beta+\beta^2)^{1/2}k}{\pi r^3 S_y(1-k^2)}\,[E(k)-k] \tag{1}$$

$$\beta = \frac{1}{3k^2}\left[2k^2-(1-k^2)\,\frac{K(k)-E(k)}{E(k)-k}\right] \tag{2}$$

Here, $K(k)$ and $E(k)$ are complete elliptic integrals of the first and second kind, where the modulus k is given by

$$k^2 = 4r^2/(l^2+4r^2) \tag{3}$$

Equation (1) can be written as

$$m_{st}' = \frac{4M^2\rho_{st}}{\pi S_y}\,\frac{1}{r^3}\,f_1\left(\frac{l}{r}\right) \tag{4}$$

The magnetic field at the center of the solenoid is denoted by H

$$H = 4M/r^2(l^2+4r^2)^{1/2} \tag{5}$$

where

$$M = \pi r^2 NI \tag{6}$$

and NI is the number of ampere turns. To prevent the superconducting wire from transforming to the "normal" state, the current I was chosen so that the magnetic field at the windings is less than the critical field. This poses a constraint on H, the field at the center of the solenoid. For our calculations we assume $H = 50{,}000$ gauss, and the magnetic dipole moment will be given in units of gauss-cubic centimeters.

By making proper choice of units, the equations defining mass and solenoidal dimensions, viz. (4–6), can be made independent of the magnetic field parameters (H, M). Now, m_{st}' can be expressed in units of λ, and r and l in units of R_0, where

$$\begin{aligned}\lambda &= 2MH\rho_{st}/\pi S_y = 7.830\times10^{-6}M\ \text{g}\\ R_0 &= 2M/H = 3.420\times10^{-2}M^{1/3}\ \text{cm}\end{aligned} \tag{7a}$$

Here

$$M = \frac{cp}{e}\left[\frac{4\pi}{3}\,f(\bar{r})\right]^{-2/3} V^{2/3} = 1.291\times10^{9}V^{2/3}\ \text{gauss-cm}^3 \tag{7b}$$

where V is measured in cubic meters. The expression for M as given previously is derived in Ref. 5 for the case where $\bar{r} = 1$, where V represents the volume of the inner forbidden region for an ideal magnetic dipole. When the mass of the structural containment shell is given in units of λ, we shall designate it by the symbol M_{st}' rather than m_{st}'; likewise when r and l are expressed in units of R_0, they are referred to as R and L, respectively. Thus, in units of λ and R_0, Eq. (4) becomes

$$M_{st}' = (1/R^3)f_1(L/R) \tag{8}$$

where now

$$L = 2R[(1/R^6)-1]^{1/2} \tag{9}$$

This procedure enables us to make an optimum choice of the dimensions of the superconducting coil in the following manner: Plots of M_{st}' vs R and L vs R based on these equations are shown in Fig. 1. Observe that M_{st}' has a minimum at $1.608\,\lambda$, corresponding to $R = 0.880R_0$ and $L = 1.08R_0$, and that from (7) we recall that λ and R_0 are functions of M alone. It is important to emphasize that the minimum value of M_{st}' obtained by this analysis is independent of (H, M) and is therefore true for all values of H and M.

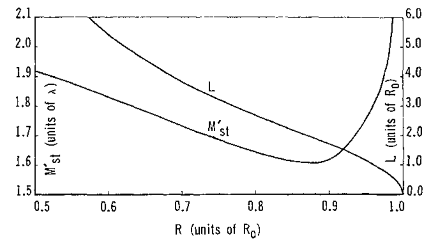

Fig. 1 Curves for optimizing the mass of the containment structure of the superconducting solenoid in terms of the dimensions of the solenoid.

Our optimization procedure consists of first selecting the magnitude of the volume to be shielded for a given maximum electron energy and then finding the corresponding magnetic moment for constant values of H. This determines λ and R_0 and thus $[M_{st}']_{\min}$; hence, $[m_{st}']_{\min}$ is determined. A plot of magnetic moment vs shielded volume for a maximum electron energy of 10 Mev appropriate for calculations pertaining to shielding against electrons of the artificial radiation belt is shown in Fig. 2. Here, the toroidally shaped forbidden volume of the superconducting solenoid (approximated by a point dipole) is compared with one of equal magnitude enclosed by a two-layer composite spherical shell of material. The outer layer consists of aluminum sufficiently thick to stop 10-Mev electrons, and the inner layer is lead of thickness specified so as to yield a given skin dose rate for a given incident electron flux. Note the curve $\mathfrak{D}_{\max}$, maximum skin dose rate, which gives the mass of material shielding for zero thickness of lead. This curve lies a full order-of-magnitude higher than the corresponding magnetic shielding mass curve. A curve showing the forbidden volume as a function of the dipole moment (M) of the solenoid is also shown for reference.

Finally, the containment structure can be modified leading to an appreciable reduction in mass.[4] A cylindrical shell at the windings counteracts the axial compression, and a series of disks inside the cylinder supports the radial forces. A correction factor is to be applied to our previous mass calculation to obtain the mass $[m_{st}]_{\min}$ for the new configuration. We have

$$[m_{st}]_{\min} = \frac{1+2(S_L/S_t)}{2(S_y/S_t)}\,[m_{st}']_{\min} \tag{10}$$

which is, in our case,

$$[m_{st}]_{\min} = 0.706\,[m_{st}']_{\min} \tag{11}$$

The mass of the wire is

$$m_w = 8.883\times10^{-6}Nr\ \text{kg} \tag{12}$$

where r is given in centimeters and N is determined from Eqs. (5) and (6).

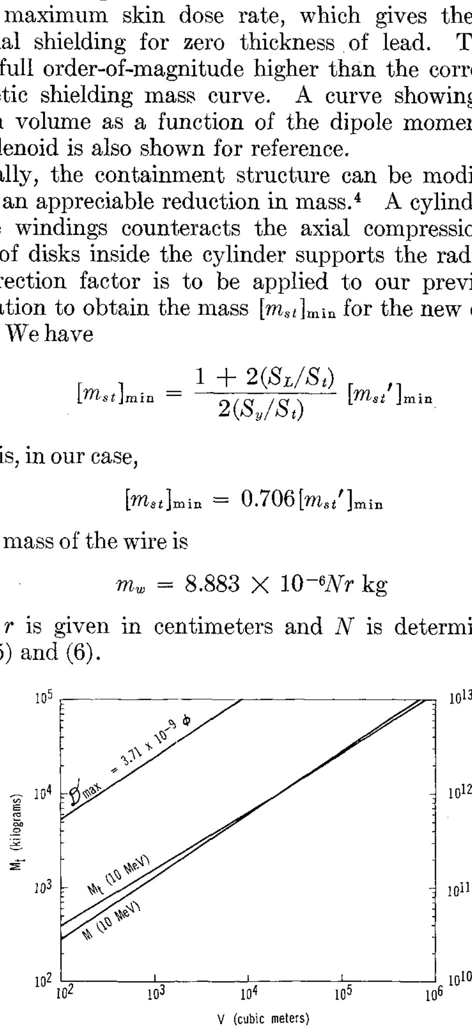

Fig. 2 Relative shielding masses M_t required to protect equal volumes of space V from electrons of the artificial radiation belt: first by magnetic dipole shielding and then by material shielding.

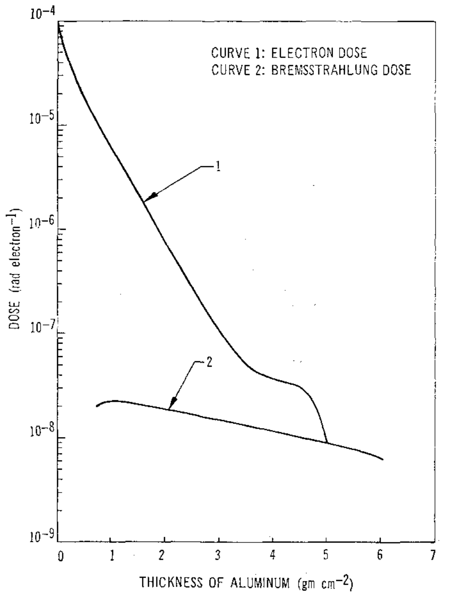

Fig. 3 Relative doses for electrons stopping in aluminum and for their attendant bremsstrahlung production, as a function of aluminum thickness, where 1 g-cm^{-2} of aluminum corresponds to 1.924×10^{-2} m.

The over-all mass of the containment structure and the windings is then given by

$$M_t = [m_{st}]_{\min} + m_w \tag{13}$$

A graph of shielded volume V as a function of over-all magnetic system mass M_t is given in Fig. 2 and will be referred to later in order to make a comparison with material shielding.

Material Shielding Mass

The bremsstrahlung skin dose rate on tissue will be the criterion utilized for calculating the mass of material shielding required for protection against electrons of the artificial radiation belt. In our model, a small sphere of tissue is placed at the center of a spherical volume protected by a two-layer composite shielding wall. The outer layer consists of low-Z material of thickness equal to the practical range of the most energetic fission-spectrum electrons (R_p); the inner layer consists of a high-Z material to attenuate the bremsstrahlung produced in the outer layer. The thickness of the inner layer is chosen so as to allow a given dose-rate for a specified average electron flux impinging on the vehicle. In our calculations we have selected aluminum for the outer layer, lead for the inner layer, and we assume that 10 Mev is the cutoff energy for electrons of the fission spectrum (14). Figure 3 illustrates the importance of completely stopping the electrons in the aluminum. The data for this graph was kindly provided by Fortney[6] and is based on the assumption of isotropic bremsstrahlung production and a specified transmission factor.[8] Observe that at given thicknesses of aluminum the electron dose is much larger than the bremsstrahlung dose. This indicates the importance of stopping the electrons completely.

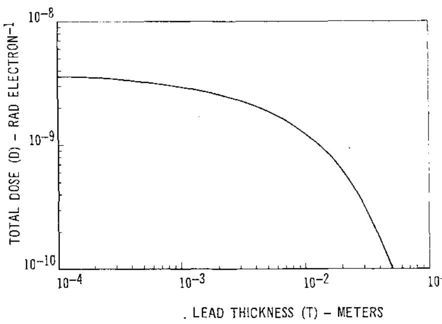

Fig. 4 Skin dose from bremsstrahlung produced by a normalized fission spectrum of electrons in aluminum and attenuated by different thicknesses of lead, where 1 g-cm^{-2} of lead is equivalent to 8.81×10^{-4} m.

We present pertinent formulas whereby a large class of shielding problems can be solved with relative ease. These results can be applied wherever the following two assumptions are satisfied:

1) The electron spectrum is given by Ref. 1:

$$y_0(E) = 3.88 \exp[-0.575E - 0.055E^2],\ E \text{ in Mev} \tag{14}$$

2) The omnidirectional electron flux encountered by the vehicle for any single trajectory is the same at all points along the trajectory within the anomaly of the artificial radiation belt. Outside this anomaly, the vehicle is in a radiation-free environment.

Implicit in our calculations is the assumption that the electron flux encountered by the vehicle is large enough to produce a significant bremsstrahlung dose rate at the surface of the tissue receiver. Our model will not give optimum shielding when this dose rate is very small.

The normalized bremsstrahlung dose from electrons of the artificial radiation belt is given by

$$D_0(T, E_0) = \int_0^{E_0} dE E_\gamma(E) \int_0^{t_m} \gamma(E_0, E, t + T + R_p) \times e^{-\sigma(E)t}\, dt \tag{15}$$

where

$\gamma(E_0, E, t + T + R_p)$ = the number of photons with energies in the interval E to $E + dE$ at a point on the inner surface
E_γ = the dose conversion factor
$\sigma(E)$ = the absorption coefficient in tissue
E_0 = the maximum energy of the fission spectrum which we take as 10 Mev
T = the thickness of the lead layer
t_m = 4.76×10^{-3} m

Since we are dealing with an omnidirectional flux, $D_0/4\pi$ is the flux per steradian and $D = 2\pi(D_0/4\pi)$ is the corresponding skin dose. That is,

$$D = D_0/2 \tag{16}$$

Equation (16) has been evaluated for a number of T-values. The results are shown graphically in Fig. 4 and can be expressed analytically to a good approximation by

$$D = a_0 e^{-\sigma_0 T} \qquad T > 3.0 \times 10^{-2}\ \text{m} \tag{17}$$

where

$$\log_{10} a_0 = -8.70043 \tag{18}$$

$$1/\sigma_0 = 43.29 \tag{19}$$

Now, if ϕ is the flux per hour of mission time encountered by the vehicle, and $\mathfrak{D}$ is the dose per hour for the model shield of lead thickness T, then we have

$$T = -0.3318 + 0.0381 \log_{10}(\phi/\mathfrak{D})\text{m} \tag{20}$$

We may now compute the total mass of material shielding as a function of shielded volume ($V > 10^{0.6}$ m^3), where

$$M_t = M_t' + a_1 T + a_2 T^2 + a_3 T^3 \text{kg} \tag{21}$$

and the parameters M_t', a_1, a_2, and a_3 are defined below. Numerical values correspond to our choice of lead and aluminum and an R_p value for 10-Mev electrons in aluminum of 0.01927 m (see Ref. 7):

$$M_t' = (4\pi/3)[(R_p + R)^3 - R^3]\rho_{Al} = 10^{2.444}V^{0.6557}$$
$$a_1 = 4\pi[R_p(R_p + 2R)\rho_{Al} + R^2\rho_{Pb}] = 10^{4.7627}V^{0.6619}$$
$$a_2 = 4\pi[R_p\rho_{Al} + R\rho_{Pb}] = 10^{4.9611}V^{1/3} \quad (21a)$$
$$a_3 = (4\pi/3)\rho_{Pb} = 7.6 \times 10^7$$

From (20–21a) we obtain

$$\log_{10}[M_t - C_1(\phi/\mathfrak{D})] = C_2(\phi/\mathfrak{D}) + 0.6619 \log_{10}V \quad (22)$$

where

$$C_1(\phi/\mathfrak{D}) = 4.203 \times 10^3[-8.7 + \log_{10}(\phi/\mathfrak{D})]^3$$
$$C_2(\phi/\mathfrak{D}) = 2.1818 + \log_{10}[-8.7 + \log_{10}(\phi/\mathfrak{D})] \quad (22a)$$

The maximum dose rate corresponds to the case $T = 0$, that is,

$$\mathfrak{D}_{max} = 3.71 \times 10^{-9}\phi \text{ rad/hr}$$

In Fig. 2 we have plotted M_t vs V for $\mathfrak{D} = \mathfrak{D}_{max}$ in order to compare with magnetic shielding. Observe that the magnetic mass is lower than the material mass by an order of magnitude even for this extreme case.

Material Shielding Examples

In Table 1 we list the values of the constants C_2 and $\log_{10}C_1$, of (22a) for four representative electron fluxes for each of three representative bremsstrahlung skin dose rates; then from Eq. (22) we can calculate masses of shielding required for a given shielded volume. The electron fluxes ϕ_i (i = 1–4) listed below correspond, respectively, to computer calculated average flux values for practical satellite orbits of 200, 400, 600, and 800 km alt inclined at 30° to the earth's equator.[8, 9] These flux value averages are good for mission times equal to or greater than 100 hr and become even more accurate for longer missions.

The dose rates were obtained by taking the NASA-specified skin doses[10] listed below:

G_1 = 125 rad (short mission maximum allowable skin dose)
G_2 = 233 rad (maximum allowable skin dose for a mission of 1 yr)
G_3 = 2282 rad (maximum allowable skin dose for a mission of 5 yr)

and performing the prescribed operations:

$$\mathfrak{D}_1 = G_1/(125 \text{ hr}) = 1(\text{rad/hr})$$
$$\mathfrak{D}_2 = G_2/(8760 \text{ hr}) = \tfrac{1}{35}\text{rad/hr})$$
$$\mathfrak{D}_3 = G_3/(43{,}800 \text{ hr}) = \tfrac{1}{20}(\text{rad/hr})$$

The corresponding masses of material shielding are given by substituting these values of C_1, C_2 in Eq. (22).

Concluding Remarks

We have used calculations pertaining to an ideal dipole to approximate the shielding effects of a superconducting solenoid against electrons of the artificial radiation belt. Our method for optimizing the mass of the superconducting solenoid and its containment shell permits us to avoid analyzing all possible systems with different r and l values. This in turn has allowed great simplification of the mathematical form in which the results are expressed. To estimate material shielding requirements against bremsstrahlung, care was taken to properly account for ionization, bremsstrahlung, and Compton scattering (using a transmission factor)[11] by electrons in aluminum. However, only skin doses on a small sphere of tissue were considered to be within the scope of the present work. The combination of aluminum and lead was considered as not only practical but also as having superior shielding characteristics. Lastly, a significant reduction in the magnetic shielding mass seems possible by integrating the containment shell with the walls of the space vehicle.

Our results indicate a significantly lower mass for the magnetic shield. However, against low electron fluxes, where bremsstrahlung shielding may not be required and the electrons need only be partly stopped, the advantage may be with material shielding. The greater reliability of material shielding will also count in its favor.

Table 1[a] Values of C_2 and $\log_{10} C_1$

	ϕ_1 = 9.3×10^8	ϕ_2 = 1.1×10^{10}	ϕ_3 = 5.3×10^{10}	ϕ_4 = 1.1×10^{10}
$D_1 = 1$	1.6905 (1.5375)[b]	4.0056 (2.3092)	4.5420 (2.4880)	4.7310 (2.5510)
$D_2 = \frac{1}{35}$	4.3986 (2.4402)	5.0040 (2.6420)	5.2806 (2.7342)	5.3916 (2.7712)
$D_3 = \frac{1}{20}$	4.2111 (2.3777)	4.8891 (2.6037)	5.1888 (2.7036)	5.3073 (2.7431)

[a] The constants C_2 and $\log_{10}C_1$ are given in the body oft he table for given paired values of $\mathfrak{D}$ and ϕ. Here, $\mathfrak{D}$ is measured in units of (rad-hr^{-1}) and ϕ in units of (electrons-cm^{-2}-hr^{-1}).
[b] Numbers in parenthesis are values of C_2.

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